Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 51315

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except B.E. Marine Engineering))

(Also Common to PTMA 3151-Matrices and calculus for B.E. (Part-Time) First Semester-All Branches-Regulations 2023)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is an eigenvalue of a matrix A, then prove that λ^2 is an eigenvalue of A^2.

2. If $x = [-1, 0, 1]^T$ is the eigenvector of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, then find the corresponding eigen value.

3. Sketch the graph of the function $f(x) = 2.0 - 0.4x$ and find the domain of the function.

4. Differentiate $y = x\tan(\sqrt{x})$ with respect to x.

5. Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.

6. If $u = x - y, v = y - z, w = z - x$, then find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

7. What is wrong with the equation $\int_{-2}^{1}\left[\frac{1}{x^4}\right]dx = \int_{-2}^{1}\left[x^{-4}\right]dx = \left[\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$.

8. Evaluate $\int_{-1}^{1}\left[\frac{\tan x}{1 + x^2 + x^4}\right]dx$ by using the concept of odd and even functions.

9. Evaluate $\int_1^2 \int_0^{x^2} [x] dy\, dx$.

10. Write the integral equation for the regions $x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1$ by triple integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2 x_3 + 2x_3 x_1 - 2x_1 x_2$ to a canonical form by orthogonal reduction. (16)

12. (a) (i) Find the value of $\lim_{x \to 2} \left[\frac{x^2 - 2}{x^3 - 3x + 5} \right]^2$. (6)

(ii) Find the local maximum and minimum values of the function $f(x) = x + 2\sin x$ in the interval $0 \leq x \leq 2\pi$. (10)

Or

(b) (i) Find an equation of the tangent line to the curve $y = \frac{e^x}{(1+x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = \log\left[x^2 + x + 1\right]$ in the interval $[-1, 1]$. (8)

13. (a) (i) If $u = \log\left[x^2 + y^2 + z^2\right]$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$? (8)

(ii) The temperature at any point (x,y,z) in space is given by $T = 400\,xyz^2$. Find the maximum temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (8)

Or

(b) (i) Expand $f(x,y) = e^{x+y}$ about the point $(0,0)$ in powers of x and y upto third degree terms by using Taylor's series. (8)

(ii) Find the maxima and minima for the given function $f(x,y) = x^3 y^2 [1 - x - y]$. (8)

14. (a) (i) Evaluate $\int x^2 e^x \, dx$ by using integration by parts. (8)

(ii) Evaluate the integral $\int \sin^4 x \, dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{a^2 - x^2} \, dx$. (8)

(ii) Evaluate $\int \frac{1}{\left(x^2 - a^2\right)} dx$ by using partial fraction. (8)

15. (a) (i) Evaluate $\int_0^{\pi/2} \int_0^{\sin\theta} [r] \, d\theta \, dr$. (8)

(ii) Change the order of integration in

$\int_0^a \int_x^a \left[x^2 + y^2\right] dy \, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\iint [xy] \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 3^2$ by using triple integration. (8)

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